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Leading coefficient test

2. If the degree of the polynomial (n) is odd: – If the leading coefficient (a_n) is positive, then as x approaches positive infinity, the function f(x) approaches positive infinity, and as x approaches negative infinity, the function f(x) approaches negative infinity. – If the leading coefficient (a_n) is negative, then as x approaches positive infinity, the function f(x) approaches negative infinity, and as x approaches negative infinity, the function f(x) approaches positive infinity. To illustrate this, let's consider an example. Suppose we have the polynomial function: f(x) = 3x^4 – 2x^3 + 5x^2 – 7x + 1 In this case, the leading term is 3x^4 with a leading coefficient of 3. Since the degree of the polynomial is even, and the leading coefficient is positive, the end behavior of the function f(x) is as follows: – As x approaches positive or negative infinity, f(x) also approaches positive infinity. Using the leading term test can help us understand the general behavior of a polynomial function without having to draw the entire graph. More Answers: Understanding Extraneous Solutions in Mathematics: Definition, Examples, and Importance Understanding Horizontal Asymptotes: Determining and Analyzing Functions in Mathematics Understanding the Significance of the Leading Coefficient in Polynomial Functions: A Guide Error 403 The request cannot be completed because you have exceeded your quota. : quotaExceeded Whether the graph of a polynomial rises or falls can be determined by the Leading Coefficient Tests. P(x) = anxn + an-1xn-1 +..... a1x + a0In the above polynomial, n is the degree and an is the leading coefficient. Case End Behavior of Graph When n is odd and an is positive Graph falls to the left and rises to the right When n is odd and an is negative Graph rises to the left and falls to the right When n is even and an is positive Graph rises to the left and right When n is even and an is negative Graph falls to the left and right Example 1 :Find the right-hand and left-hand behaviors of the graph of f(x) = x5 + 2x3 - 3x + 5 Solution : Because the degree is odd and the leading coefficient is positive, the graph falls to the left and rises to the right as shown in the figure. Example 2 : Determine the end behavior of the graph of the polynomial function below using Leading Coefficient Test. P(x) = -x3 + 5xSolution : Because the degree is odd and the leading coefficient is negative, the graph rises to the left and falls to the right as shown in the figure. Example 3 :Determine the end behavior of the graph of the polynomial function below using Leading Coefficient Test. P(x) = 2x2 - 2Solution : Because the degree is even and the leading coefficient is positive, the graph rises to the left and right as shown in the figure. Example 4 :Determine the end behavior of the graph of the polynomial function below using Leading Coefficient Test. P(x) = -x2 + 1Solution : Because the degree is even and the leading coefficient is negative, the graph falls to the left and right as shown in the figure. Kindly mail your feedback to v4formath@gmail.comWe always appreciate your feedback. ©All rights reserved. onlinemath4all.com ©2025 Varsity Tutors - A Nerdy Company. 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Columbia University.For press inquiries please contact press@varsitytutors.com LEARNING OBJECTIVES By the end of this lesson, you will be able to: Identify power functions. Identify end behavior of power functions. Identify polynomial functions. Identify the degree and leading coefficient of polynomial functions. Figure 1. (credit: Jason Bay, Flickr) Suppose a certain species of bird thrives on a small island. Its population over the last few years is shown below. Year 2009 2010 2011 2012 2013 Bird Population 800 897 992 1,083 1,169 The population can be estimated using the function [latex]P\left(t\right)=-0.3{t}^{3}+97t+800[/latex], where [latex]P\left(t\right)[/latex] represents the bird population on the island t years after 2009. We can use this model to estimate the maximum bird population and when it will occur. We can also use this model to predict when the bird population will disappear from the island. In this section, we will examine functions that we can use to estimate and predict these types of changes. Identify power functions In order to better understand the bird problem, we need to understand a specific type of function. A power function is a function with a single term that is the product of a real number, a coefficient, and a variable raised to a fixed real number. (A number that multiplies a variable raised to an exponent is known as a coefficient.) As an example, consider functions for area or volume. The function for the area of a circle with radius r is [latex]A\left(r\right)=\pi {r}^{2}[/latex] and the function for the volume of a sphere with radius r is [latex]V\left(r\right)=\frac{4}{3}\pi {r}^{3}[/latex] Both of these are examples of power functions because they consist of a coefficient, [latex]\pi [/latex] or [latex]\frac{4}{3}\pi [/latex], multiplied by a variable r raised to a power. A power function is a function that can be represented in the form [latex]f\left(x\right)=k{x}^{p}[/latex] where k and p are real numbers, and k is known as the coefficient. Is [latex]f\left(x\right)={2}^{x}[/latex] a power function? No. A power function contains a variable base raised to a fixed power. This function has a constant base raised to a variable power. This is called an exponential function, not a power function. Which of the following functions are power functions? [latex]\begin{cases}f\left(x\right)=1\hfill & \text{Constant function}\hfill \\ f\left(x\right)=x\hfill & \text{Identity function}\hfill \\ f\left(x\right)={x}^{2}\hfill & \text{Quadratic}\text{ }\text{ function}\hfill \\ f\left(x\right)={x}^{3}\hfill & \text{Cubic function}\hfill \\ f\left(x\right)=\frac{1}{x} \hfill & \text{Reciprocal function}\hfill \\ f\left(x\right)=\frac{1}{{x}^{2}}\hfill & \text{Reciprocal squared function}\hfill \\ f\left(x\right)=\sqrt{x}\hfill & \text{Square root function}\hfill \\ f\left(x\right)=\sqrt[3]{x}\hfill & \text{Cube root function}\hfill \end{cases}[/latex] All of the listed functions are power functions. The constant and identity functions are power functions because they can be written as [latex]f\left(x\right)={x}^{0}[/latex] and [latex]f\left(x\right)={x}^{1}[/latex] respectively. The quadratic and cubic functions are power functions with whole number powers [latex]f\left(x\right)={x}^{2}[/latex] and [latex]f\left(x\right)={x}^{3}[/latex]. The reciprocal and reciprocal squared functions are power functions with negative whole number powers because they can be written as [latex]f\left(x\right)={x}^{-1}[/latex] and [latex]f\left(x\right)={x}^{-2}[/latex]. The square and cube root functions are power functions with fractional powers because they can be written as [latex]f\left(x\right)={x}^{1/2}[/latex] or [latex]f\left(x\right)={x}^{1/3}[/latex]. Identify end behavior of power functions Figure 2 shows the graphs of [latex]f\left(x\right)={x}^{2},g\left(x\right)={x}^{4}[/latex] and [latex]\text{and}h\left(x\right)={x}^{6}[/latex], which are all power functions with even, whole-number powers. Notice that these graphs have similar shapes, very much like that of the quadratic function in the toolkit. However, as the power increases, the graphs flatten somewhat near the origin and become steeper away from the origin. Figure 2. Even-power functions To describe the behavior as numbers become larger and larger, we use the idea of infinity. We use the symbol [latex]\infty[/latex] for positive infinity and [latex]-\infty[/latex] for negative infinity. When we say that "x approaches infinity," which can be symbolically written as [latex]x\to \infty\\ [/latex], we are describing a behavior; we are saying that x is increasing without bound. With the even-power function, as the input increases or decreases without bound, the output values become very large, positive numbers. Equivalently, we could describe this behavior by saying that as [latex]x[/latex] approaches positive or negative infinity, the [latex]f\left(x\right)[/latex] values increase without bound. In symbolic form, we could write [latex]\text{as }x\to \pm \infty , f\left(x\right)\to \infty\\ [/latex] Figure 3 shows the graphs of [latex]f\left(x\right)={x}^{3},g\left(x\right)={x}^{5},\text{and}h\left(x\right)={x}^{7}[/latex], which are all power functions with odd, whole-number powers. Notice that these graphs look similar to the cubic function in the toolkit. Again, as the power increases, the graphs flatten near the origin and become steeper away from the origin. Figure 3. Odd-power function These examples illustrate that functions of the form [latex]f\left(x\right)={x}^{n}[/latex] reveal symmetry of one kind or another. First, in Figure 2 we see that even functions of the form [latex]f\left(x\right)={x}^{n}\text{, }n\text{ even,}[/latex] are symmetric about the y-axis. In Figure 3 we see that odd functions of the form [latex]f\left(x\right)={x}^{n}\text{, }n\text{ odd,}[/latex] are symmetric about the origin. For these odd power functions, as x approaches negative infinity, [latex]f\left(x\right)[/latex] decreases without bound. As x approaches positive infinity, [latex]f\left(x\right)[/latex] increases without bound. In symbolic form we write [latex]\begin{cases}\text{as } x\to -\infty , f\left(x\right)\to -\infty \\ \text{as } x\to \infty , f\left(x\right)\to \infty \end{cases}[/latex] The behavior of the graph of a function as the input values get very small ([latex]x\to -\infty[/latex]) and get very large ([latex]x\to \infty[/latex]) is referred to as the end behavior of the function. We can use words or symbols to describe end behavior. The table below shows the end behavior of power functions in the form [latex]f\left(x\right)=k{x}^{n}[/latex] where [latex]n[/latex] is a non-negative integer depending on the power and the constant. Even power Odd power Positive constant k > 0 Negative constant k < 0 How To: Given a power function [latex]f\left(x\right)=k{x}^{n}[/latex] where n is a non-negative integer, identify the end behavior. Determine whether the power is even or odd. Determine whether the constant is positive or negative. Use Figure 4 to identify the end behavior. Describe the end behavior of the graph of [latex]f\left(x\right)={x}^{8}[/latex]. The coefficient is 1 (positive) and the exponent of the power function is 8 (an even number). As x approaches infinity, the output (value of [latex]f\left(x\right)[/latex]) increases without bound. We write as [latex]x\to \infty , f\left(x\right)\to \infty [/latex]. As x approaches negative infinity, the output increases without bound. In symbolic form, as [latex]x\to -\infty , f\left(x\right)\to \infty\\ [/latex]. We can graphically represent the function as shown in Figure 5. Figure 4 Describe the end behavior of the graph of [latex]f\left(x\right)=-{x}^{9}[/latex]. The exponent of the power function is 9 (an odd number). Because the coefficient is –1 (negative), the graph is the reflection about the x-axis of the graph of [latex]f\left(x\right)={x}^{9}[/latex]. The graph shows that as x approaches infinity, the output decreases without bound. As x approaches negative infinity, the output increases without bound. In symbolic form, we would write Figure 5. [latex]\begin{cases}\text{as } x\to -\infty , f\left(x\right)\to \infty \\ \text{as } x\to \infty , f\left(x\right)\to -\infty \end{cases}[/latex] Describe in words and symbols the end behavior of [latex]f\left(x\right)=-5{x}^{4}[/latex]. Solution Identify polynomial functions An oil pipeline bursts in the Gulf of Mexico, causing an oil slick in a roughly circular shape. The slick is currently 24 miles in radius, but that radius is increasing by 8 miles each week. We want to write a formula for the area covered by the oil slick by combining two functions. The radius r of the spill depends on the number of weeks w that have passed. This relationship is linear. [latex]r\left(w\right)=24+8w[/latex] We can combine this with the formula for the area A of a circle. [latex]A\left(r\right)=\pi {r}^{2}[/latex] Composing these functions gives a formula for the area in terms of weeks. [latex]\begin{cases}A\left(w\right)=A\left(r\left(w\right)\right)\\ =A\left(24+8w\right)\\ =\pi {\left(24+8w\right)}^{2}\end{cases}[/latex] Multiplying gives the formula. [latex]A\left(w\right)=576\pi +384\pi w+64\pi {w}^{2}[/latex] This formula is an example of a polynomial function. A polynomial function consists of either zero or the sum of a finite number of non-zero terms, each of which is a product of a number, called the coefficient of the term, and a variable raised to a non-negative integer power. Let n be a non-negative integer. A polynomial function is a function that can be written in the form [latex]f\left(x\right)={a}_{n}{x}^{n}+\dots+{a}_{2}{x}^{2}+{a}_{1}x+{a}_{0}[/latex] This is called the general form of a polynomial function. Each [latex]{a}_{i}[/latex] is a coefficient and can be any real number. Each product [latex]{a}_{i}{x}^{i}[/latex] is a term of a polynomial function. Which of the following are polynomial functions? [latex]\begin{cases}f\left(x\right)=2{x}^{3}\cdot 3x+4\hfill \\ g\left(x\right)=-x\left({x}^{2}-4\right)\hfill \\ h\left(x\right)=5\sqrt{x}+2\hfill \end{cases}[/latex] The first two functions are examples of polynomial functions because they can be written in the form [latex]f\left(x\right)={a}_{n}{x}^{n}+\dots+{a}_{2}{x}^{2}+{a}_{1}x+{a}_{0}[/latex], where the powers are non-negative integers and the coefficients are real numbers. [latex]f\left(x\right)[/latex] can be written as [latex]f\left(x\right)=6{x}^{4}+4[/latex]. [latex]g\left(x\right)[/latex] can be written as [latex]g\left(x\right)=-{x}^{3}+4x[/latex]. [latex]h\left(x\right)[/latex] cannot be written in this form and is therefore not a polynomial function. Identify the degree and leading coefficient of polynomial functions Because of the form of a polynomial function, we can see an infinite variety in the number of terms and the power of the variable. Although the order of the terms in the polynomial function is not important for performing operations, we typically arrange the terms in descending order of power, or in general form. The degree of the polynomial is the highest power of the variable that occurs in the polynomial; it is the power of the first variable if the function is in general form. The leading term is the term containing the highest power of the variable, or the term with the highest degree. The leading coefficient is the coefficient of the leading term. Figure 6 We often rearrange polynomials so that the powers are descending. When a polynomial is written in this way, we say that it is in general form. How To: Given a polynomial function, identify the degree and leading coefficient. Find the highest power of x to determine the degree function. Identify the term containing the highest power of x to find the leading term. Identify the coefficient of the leading term. Identify the degree, leading term, and leading coefficient of the following polynomial functions. [latex]\begin{cases} f\left(x\right)=3+2{x}^{2}-4{x}^{3} \\ g\left(t\right)=5{t}^{5}-2{t}^{3}+7t\\ h\left(p\right)=6p-{p}^{3}-2\end{cases}[/latex] For the function [latex]f\left(x\right)[/latex], the highest power of x is 3, so the degree is 3. The leading term is the term containing that degree, [latex]-4{x}^{3}[/latex]. The leading coefficient is the coefficient of that term, –4. For the function [latex]g\left(t\right)[/latex], the highest power of t is 5, so the degree is 5. The leading term is the term containing that degree, [latex]5{t}^{5}[/latex]. The leading coefficient is the coefficient of that term, 5. For the function [latex]h\left(p\right)[/latex], the highest power of p is 3, so the degree is 3. The leading term is the term containing that degree, [latex]-{p}^{3}[/latex]; the leading coefficient is the coefficient of that term, –1. Identify the degree, leading term, and leading coefficient of the polynomial [latex]f\left(x\right)=4{x}^{2}-{x}^{6}+2x - 6[/latex]. Solution Knowing the degree of a polynomial function is useful in helping us predict its end behavior. To determine its end behavior, look at the leading term of the polynomial function. Because the power of the leading term is the highest, that term will grow significantly faster than the other terms as x gets very large or very small, so its behavior will dominate the graph. For any polynomial, the end behavior of the polynomial will match the end behavior of the term of highest degree. Polynomial Function Leading Term Graph of Polynomial Function [latex]f\left(x\right)=5{x}^{4}+2{x}^{3}-x - 4[/latex] [latex]5{x}^{4}[/latex] [latex]f\left(x\right)=-2{x}^{6}-{x}^{5}+3{x}^{4}+{x}^{3}[/latex] [latex]-2{x}^{6}[/latex] [latex]f\left(x\right)=3{x}^{5}-4{x}^{4}+2{x}^{2}+1[/latex] [latex]3{x}^{5}[/latex] [latex]f\left(x\right)=-6{x}^{3}+7{x}^{2}+3x+1[/latex] [latex]-6{x}^{3}[/latex] Describe the end behavior and determine a possible degree of the polynomial function in Figure 7. Figure 7 As the input values x get very large, the output values [latex]f\left(x\right)[/latex] increase without bound. As the input values x get very small, the output values [latex]f\left(x\right)[/latex] decrease without bound. We can describe the end behavior symbolically by writing [latex]\begin{cases}\text{as } x\to -\infty , f\left(x\right)\to -\infty \\ \text{as } x\to \infty , f\left(x\right)\to \infty \end{cases}[/latex] In words, we could say that as x values approach infinity, the function values approach infinity, and as x values approach negative infinity, the function values approach negative infinity. We can tell this graph has the shape of an odd degree power function that has not been reflected, so the degree of the polynomial creating this graph must be odd and the leading coefficient must be positive. Describe the end behavior, and determine a possible degree of the polynomial function in Figure 9. Figure 9 Solution Given the function [latex]f\left(x\right)=-3{x}^{2}\left(x - 1\right)\left(x+4\right)[/latex], express the function as a polynomial in general form, and determine the leading term, degree, and end behavior of the function. Obtain the general form by expanding the given expression for [latex]f\left(x\right)[/latex]. [latex]\begin{cases} f\left(x\right)=-3{x}^{2}\left(x - 1\right)\left(x+4\right)\\ \hfill =-3{x}^{2}\left({x}^{2}+3x - 4\right)\\ \hfill=-3{x}^{4}-9{x}^{3}+12{x}^{2}\end{cases}[/latex] The general form is [latex]f\left(x\right)=-3{x}^{4}-9{x}^{3}+12{x}^{2}[/latex]. The leading term is [latex]-3{x}^{4}[/latex]; therefore, the degree of the polynomial is 4. The degree is even (4) and the leading coefficient is negative (–3), so the end behavior is [latex]\begin{cases}\text{as } x\to -\infty , f\left(x\right)\to -\infty \\ \text{as } x\to \infty , f\left(x\right)\to -\infty \end{cases}[/latex] Given the function [latex]f\left(x\right)=0.2\left(x - 2\right)\left(x+1\right)\left(x - 5\right)[/latex], express the function as a polynomial in general form and determine the leading term, degree, and end behavior of the function. Solution In addition to the end behavior of polynomial functions, we are also interested in what happens in the "middle" of the function. In particular, we are interested in locations where graph behavior changes. A turning point is a point at which the function values change from increasing to decreasing or decreasing to increasing. Figure 10 We are also interested in the intercepts. As with all functions, the y-intercept is the point at which the graph intersects the vertical axis. The point corresponds to the coordinate pair in which the input value is zero. Because a polynomial is a function, only one output value corresponds to each input value so there can be only one y-intercept [latex]\left(0,{a}_{0}\right)[/latex]. The x-intercepts occur at the input values that correspond to an output value of zero. It is possible to have more than one x-intercept. A turning point of a graph is a point at which the graph changes direction from increasing to decreasing or decreasing to increasing. The y-intercept is the point at which the function has an input value of zero. The x-intercepts are the points at which the output value is zero. How To: Given a polynomial function, determine the intercepts. Determine the y-intercept by setting [latex]x=0[/latex] and finding the corresponding output value. Determine the x-intercepts by solving for the input values that yield an output value of zero. Given the polynomial function [latex]f\left(x\right)=\left(x - 2\right)\left(x+1\right)\left(x - 4\right)[/latex], written in factored form for your convenience, determine the y- and x-intercepts. The y-intercept occurs when the input is zero so substitute 0 for x. [latex]\begin{cases}f\left(0\right)=\left(0 - 2\right)\left(0+1\right)\left(0 - 4\right)\hfill \\ \text{ }=\left(-2\right)\left(1\right)\left(-4\right)\hfill \\ \text{ }=8\hfill \end{cases}[/latex] The y-intercept is (0, 8). The x-intercepts occur when the output is zero. [latex]\begin{cases}\text{ }0=\left(x - 2\right)\left(x+1\right)\left(x - 4\right)\hfill \\ x - 2=0\hfill & \hfill & \text{or}\hfill & \hfill & x+1=0\hfill & \hfill & \text{or}\hfill & \hfill & x - 4=0\hfill \\ \text{ }x=2\hfill & \hfill & \text{or}\hfill & \hfill & \text{ }x=-1\hfill & \hfill & \text{or}\hfill & \hfill & x=4 \end{cases}[/latex] The x-intercepts are [latex]\left(2,0\right),\left(-1,0\right)[/latex], and [latex]\left(4,0\right)[/latex]. We can see these intercepts on the graph of the function shown in Figure 11. Figure 11 Given the polynomial function [latex]f\left(x\right)={x}^{4}-4{x}^{2}-45[/latex], determine the y- and x-intercepts. The y-intercept occurs when the input is zero. [latex]\begin{cases} \\ f\left(0\right)={\left(0\right)}^{4}-4{\left(0\right)}^{2}-45\hfill \hfill \\ \text{ }=-45\hfill \end{cases}[/latex] The y-intercept is [latex]\left(0,-45\right)[/latex]. The x-intercepts occur when the output is zero. To determine when the output is zero, we will need to factor the polynomial. [latex]\begin{cases}f\left(x\right)={x}^{4}-4{x}^{2}-45\hfill \\ =\left({x}^{2}-9\right)\left({x}^{2}+5\right)\hfill \\ =\left(x - 3\right)\left(x+3\right)\left({x}^{2}+5\right)\hfill \end{cases}[/latex] [latex]0=\left(x - 3\right)\left(x+3\right)\left({x}^{2}+5\right)[/latex] [latex]\begin{cases}x - 3=0\hfill & \text{or}\hfill & x+3=0\hfill & \text{or}\hfill & {x}^{2}+5=0\hfill \\ \text{ }x=3\hfill & \text{or}\hfill & \text{ }x=-3\hfill & \text{or}\hfill & \text{(no real solution)}\hfill \end{cases}[/latex] The x-intercepts are [latex]\left(3,0\right)[/latex] and [latex]\left(-3,0\right)[/latex]. We can see these intercepts on the graph of the function shown in Figure 12. We can see that the function is even because [latex]f\left(x\right)=f\left(-x\right)[/latex]. Figure 12 Given the polynomial function [latex]f\left(x\right)=2{x}^{3}-6{x}^{2}-20x[/latex], determine the y- and x-intercepts. Solution The degree of a polynomial function helps us to determine the number of x-intercepts and the number of turning points. A polynomial function of nth degree is the product of n factors, so it will have at most n roots or zeros, or x-intercepts. The graph of the polynomial function of degree n must have at most n – 1 turning points. This means the graph has at most one fewer turning point than the degree of the polynomial or one fewer than the number of factors. A continuous function has no breaks in its graph: the graph can be drawn without lifting the pen from the paper. A smooth curve is a graph that has no sharp corners. The turning points of a smooth graph must always occur at rounded curves. The graphs of polynomial functions are both continuous and smooth. A polynomial of degree n will have, at most, n x-intercepts and n – 1 turning points. Without graphing the function, determine the local behavior of the function by finding the maximum number of x-intercepts and turning points for [latex]f\left(x\right)=-3{x}^{10}+4{x}^{7}-{x}^{4}+2{x}^{3}[/latex]. The polynomial has a degree of 10, so there are at most n x-intercepts and at most n – 1 turning points. Without graphing the function, determine the maximum number of x-intercepts and turning points for [latex]f\left(x\right)=108 - 13{x}^{9}-8{x}^{4}+14{x}^{12}+2{x}^{3}[/latex] Solution What can we conclude about the polynomial represented by the graph shown in the graph in Figure 13 based on its intercepts and turning points? Figure 13 Figure 14 The end behavior of the graph tells us this is the graph of an even-degree polynomial. The graph has 2 x-intercepts, suggesting a degree of 2 or greater, and 3 turning points, suggesting a degree of 4 or greater. Based on this, it would be reasonable to conclude that the degree is even and at least 4. What can we conclude about the polynomial represented by Figure 15 based on its intercepts and turning points? Figure 15 Solution Given the function [latex]f\left(x\right)=-4x\left(x+3\right)\left(x - 4\right)[/latex], determine the local behavior. The y-intercept is found by evaluating [latex]f\left(0\right)[/latex]. [latex]\begin{cases}f\left(0\right)=-4\left(0\right)\left(0+3\right)\left(0 - 4\right)\hfill \hfill \\ \text{ }=0\hfill \end{cases}[/latex] The y-intercept is [latex]\left(0,0\right)[/latex]. The x-intercepts are found by determining the zeros of the function. [latex]\begin{cases}0=-4x\left(x+3\right)\left(x - 4\right)\\ x=0\hfill & \hfill & \text{or}\hfill & \hfill & x+3=0\hfill & \hfill & \text{or}\hfill & \hfill & x - 4=0\hfill \\ x=0\hfill & \hfill & \text{or}\hfill & \hfill & \text{ }x=-3\hfill & \hfill & \text{or}\hfill & \hfill & \text{ }x=4\end{cases}[/latex] The x-intercepts are [latex]\left(0,0\right),\left(-3,0\right)[/latex], and [latex]\left(4,0\right)[/latex]. The degree is 3 so the graph has at most 2 turning points. Given the function [latex]f\left(x\right)=0.2\left(x - 2\right)\left(x+1\right)\left(x - 5\right)[/latex], determine the local behavior. Solution general form of a polynomial function [latex]f\left(x\right)={a}_{n}{x}^{n}+\dots+{a}_{2}{x}^{2}+{a}_{1}x+{a}_{0}[/latex] A power function is a variable base raised to a number power. The behavior of a graph as the input decreases beyond bound and increases beyond bound is called the end behavior. The end behavior depends on whether the power is even or odd. A polynomial function is the sum of terms, each of which consists of a transformed power function with positive whole number power. The degree of a polynomial function is the highest power of the variable that occurs in a polynomial. The term containing the highest power of the variable is called the leading term. The coefficient of the leading term is called the leading coefficient. The end behavior of a polynomial function is the same as the end behavior of the power function represented by the leading term of the function. A polynomial of degree n will have at most n x-intercepts and at most n – 1 turning points. coefficient a nonzero real number multiplied by a variable raised to an exponent continuous function a function whose graph can be drawn without lifting the pen from the paper because there are no breaks in the graph degree the highest power of the variable that occurs in a polynomial end behavior the behavior of the graph of a function as the input decreases without bound and increases without bound leading coefficient the coefficient of the leading term leading term the term containing the highest power of the variable polynomial function a function that consists of either zero or the sum of a finite number of non-zero terms, each of which is a product of a number, called the coefficient of the term, and a variable raised to a non-negative integer power. power function a function that can be represented in the form [latex]f\left(x\right)=k{x}^{p}[/latex] where k is a constant, the base is a variable, and the exponent, p, is a constant smooth curve a graph with no sharp corners term of a polynomial function any [latex]{a}_{i}{x}^{i}[/latex] of a polynomial function in the form [latex]f\left(x\right)={a}_{n}{x}^{n}+\dots+{a}_{2}{x}^{2}+{a}_{1}x+{a}_{0}[/latex] turning point the location at which the graph of a function changes direction 1. Explain the difference between the coefficient of a power function and its degree. 2. If a polynomial function is in factored form, what would be a good first step in order to determine the degree of the function? 3. In general, explain the end behavior of a power function with odd degree if the leading coefficient is positive. 4. What is the relationship between the degree of a polynomial function and the maximum number of turning points in its graph? 5. What can we conclude if, in general, the graph of a polynomial function exhibits the following end behavior? As [latex]x\to -\infty ,f\left(x\right)\to -\infty [/latex] and as [latex]x\to \infty ,f\left(x\right)\to -\infty [/latex]. For the following exercises, identify the function as a power function, a polynomial function, or neither. 6. [latex]f\left(x\right)={x}^{5}[/latex] 7. [latex]f\left(x\right)={\left({x}^{2}\right)}^{3}[/latex] 8. [latex]f\left(x\right)=x-{x}^{4}[/latex] 9. [latex]f\left(x\right)=\frac{{x}^{2}}{{x}^{2}-1}[/latex] 10. [latex]f\left(x\right)=2x\left(x+2\right){\left(x - 1\right)}^{2}[/latex] 11. [latex]f\left(x\right)={3}^{x+1}[/latex] For the following exercises, find the degree and leading coefficient for the given polynomial. 12. [latex]-3x{}^{4}[/latex] 13. [latex]7 - 2{x}^{2}[/latex] 14. [latex]-2{x}^{2}- 3{x}^{5}+ x - 6 [/latex] 15. [latex]x\left(4-{x}^{2}\right)\left(2x+1\right)[/latex] 16. [latex]{x}^{2}{\left(2x - 3\right)}^{2}[/latex] For the following exercises, determine the end behavior of the functions. 17. [latex]f\left(x\right)={x}^{4}[/latex] 18. [latex]f\left(x\right)={x}^{3}[/latex] 19. [latex]f\left(x\right)=-{x}^{4}[/latex] 20. [latex]f\left(x\right)=-{x}^{9}[/latex] 21. [latex]f\left(x\right)=-2{x}^{4}- 3{x}^{2}+ x - 1[/latex] 22. [latex]f\left(x\right)=3{x}^{2}+ x - 2[/latex] 23. [latex]f\left(x\right)={x}^{2}\left(2{x}^{3}-x+1\right)[/latex] 24. [latex]f\left(x\right)={\left(2-x\right)}^{7}[/latex] For the following exercises, find the intercepts of the functions. 25. [latex]f\left(t\right)=2\left(t - 1\right)\left(t+2\right)\left(t - 3\right)[/latex] 26. [latex]g\left(n\right)=-2\left(3n - 1\right)\left(2n+1\right)[/latex] 27. [latex]f\left(x\right)={x}^{4}-16[/latex] 28. [latex]f\left(x\right)={x}^{3}+27[/latex] 29. [latex]f\left(x\right)=x\left({x}^{2}-2x - 8\right)[/latex] 30. [latex]f\left(x\right)=\left(x+3\right)\left(4{x}^{2}-1\right)[/latex] For the following exercises, determine the least possible degree of the polynomial function shown. 31. 32. 33. 34. 35. 36. 37. 38. For the following exercises, determine whether the graph of the function provided is a graph of a polynomial function. If so, determine the number of turning points and the least possible degree for the function. 39. 40. 41. 42. 43. 44. 45. For the following exercises, make a table to confirm the end behavior of the function. 46. [latex]f\left(x\right)=-{x}^{3}[/latex] 47. [latex]f\left(x\right)={x}^{4}-5{x}^{2}[/latex] 48. [latex]f\left(x\right)={x}^{2}{\left(1-x\right)}^{2}[/latex] 49. [latex]f\left(x\right)=\left(x - 1\right)\left(x - 2\right)\left(3-x\right)[/latex] 50. [latex]f\left(x\right)=\frac{{x}^{5}}{10}-{x}^{4}[/latex] For the following exercises, graph the polynomial functions using a calculator. Based on the graph, determine the intercepts and the end behavior. 51. [latex]f\left(x\right)={x}^{3}\left(x - 2\right)[/latex] 52. [latex]f\left(x\right)=x\left(x - 3\right)\left(x+3\right)[/latex] 53. [latex]f\left(x\right)=x\left(14 - 2x\right)\left(10 - 2x\right)[/latex] 54. [latex]f\left(x\right)=x\left(14 - 2x\right){\left(10 - 2x\right)}^{2}[/latex] 55. [latex]f\left(x\right)={x}^{3}-16x[/latex] 56. [latex]f\left(x\right)={x}^{3}-27[/latex] 57. [latex]f\left(x\right)={x}^{4}-81[/latex] 58. [latex]f\left(x\right)=-{x}^{3}+{x}^{2}+2x[/latex] 59. [latex]f\left(x\right)={x}^{3}-2{x}^{2}-15x[/latex] 60. [latex]f\left(x\right)={x}^{3}-0.01x[/latex] For the following exercises, use the information about the graph of a polynomial function to determine the function. Assume the leading coefficient is 1 or –1. There may be more than one correct answer. 61. The y-intercept is [latex]\left(0,-4\right)[/latex]. The x-intercepts are [latex]\left(-2,0\right),\left(2,0\right)[/latex]. Degree is 2. End behavior: [latex]\text{as }x\to -\infty ,f\left(x\right)\to \infty ,\text{ as }x\to \infty ,f\left(x\right)\to \infty [/latex]. 62. The y-intercept is [latex]\left(0,9\right)[/latex]. The x-intercepts are [latex]\left(-3,0\right),\left(3,0\right)[/latex]. Degree is 2. End behavior: [latex]\text{as }x\to -\infty ,f\left(x\right)\to -\infty ,\text{ as }x\to \infty ,f\left(x\right)\to -\infty\\ [/latex]. 63. The y-intercept is [latex]\left(0,0\right)[/latex]. The x-intercepts are [latex]\left(0,0\right),\left(2,0\right)[/latex]. Degree is 3. End behavior: [latex]\text{as }x\to -\infty ,f\left(x\right)\to -\infty ,\text{ as }x\to \infty ,f\left(x\right)\to \infty [/latex]. 64. The y-intercept is [latex]\left(0,1\right)[/latex]. The x-intercept is [latex]\left(1,0\right)[/latex]. Degree is 3. End behavior: [latex]\text{as }x\to -\infty ,f\left(x\right)\to \infty ,\text{ as }x\to \infty ,f\left(x\right)\to -\infty [/latex]. 65. The y-intercept is [latex]\left(0,1\right)[/latex]. There is no x-intercept. Degree is 4. End behavior: [latex]\text{as }x\to -\infty ,f\left(x\right)\to \infty ,\text{ as }x\to \infty ,f\left(x\right)\to \infty[/latex]. For the following exercises, use the written statements to construct a polynomial function that represents the required information. 66. An oil slick is expanding as a circle. The radius of the circle is increasing at the rate of 20 meters per day. Express the area of the circle as a function of d, the number of days elapsed. 67. A cube has an edge of 3 feet. The edge is increasing at the rate of 2 feet per minute. Express the volume of the cube as a function of m, the number of minutes elapsed. 68. A rectangle has a length of 10 inches and a width of 6 inches. If the length is increased by x inches and the width increased by twice that amount, express the area of the rectangle as a function of x. 69. An open box is to be constructed by cutting out square corners of x-inch sides from a piece of cardboard 8 inches by 8 inches and then folding up the sides. Express the volume of the box as a function of x. 70. A rectangle is twice as long as it is wide. Squares of side 2 feet are cut out from each corner. Then the sides are folded up to make an open box. Express the volume of the box as a function of the width (x). CC licensed content, Specific attribution