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Differential equation practice problems

Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. For problems 1 – 3 compute the differential of the given function. Page for Differential Equation practice problems. Here, a list of practice problems will be given for the course on Differential Equation (Exercise). Q1: Find the order and degree of $\left(\dfrac{d^3y}{dx^3} \right)^5+x^2y \left(\dfrac{d^4y}{dx^4} \right)^2=\sin(\ln x)$. Q2: Find the value of m for which the differential equation $\left(xy^2+mx^2y\right)dx + \left(x+y\right)x^2dy=0$ is exact and solve it for this value of m. Q3: Eliminate A and B from to form an ODE, where A and B are constants. y = e-x(A + Bx) y = A cosx + B sinx y = Aex + Be-x Q4: Evaluate $\dfrac{1}{D-2}e^{2x}$. Show the set {cos2x, sin2x} is linearly independent. Q5: Solve the following differential equations. Q5. Find the complementary function and the particular integral of [How to Find Particular Integral] (i) $\dfrac{d^2y}{dx^2}+4y= \sin^2{x}$.(ii) $\dfrac{d^2y}{dx^2}+2 \dfrac{dy}{dx}+y=x^2+e^{-x}$ (iii) $\dfrac{d^2y}{dx^2}-4 \dfrac{dy}{dx}+4y=xe^{2x}$. Q6: Using variation of parameters, solve the differential equation $\dfrac{d^2y}{dx^2}+y= \text{cosec}~x$ $\dfrac{d^2y}{dx^2}+4y= \sec 2x$ Q7: Solve the Cauchy-Euler differential equation $x^2 \dfrac{d^2y}{dx^2}-3x \dfrac{dy}{dx}+4y=\cos(\ln x)$ Q9: (a) Show the function f(x,y) = xy2 satisfies the Lipschitz condition on the rectangle |x| ≤ 1 and |y| ≤ 1. (b) Find the interval in which the initial value problem (IVP) $\dfrac{dy}{dx}=y^2, \quad y(1)=1$ has a unique solution. Q10: Practice Problems on Wronskian discussed in the class. Q1: Discuss the types of a first order PDE with examples. Q2: Solve x(y-z)p + y(z-x)q = z(x-y) using Lagrange's method. Q3: Find a complete integral of Q4: Find the values of x and y for which the partial differential equation $y\dfrac{\partial^2u}{\partial x^2} – x\dfrac{\partial^2u}{\partial y^2}$ is elliptic, hyperbolic and parabolic. Q1: Define an orthogonal trajectory for a family of curves. Find the orthogonal trajectory of the family of circles: [How to Find Orthogonal Trajectory] x2+y2 = c2 where c is an arbitrary constant. Q2: The coordinates of a moving particle are given by $x=t^2$ and $y=t^3$. Find the velocity and acceleration of the particle when $t=1$. This article is written by Dr. Tathagata Mandal, Ph.D in Mathematics from IISER Pune (Algebraic Number Theory). Thank you for visiting the website. Important Questions For Class 12 Maths Chapter 9 Differential Equations helps in preparing for CBSE Board exams. All the questions are prepared by our Maths experts. Apart from studying and practising problems on the Differential equation from the NCERT book, students shall also practice these important questions. Class 12 Maths Chapter 9 contains many formulas and methods for solving given differential equations. Students can go through with the important questions for all Maths Class 12 chapters here to revise all the chapters and score good marks in the CBSE final examination. Important Questions & Answers For Class 12 Maths Chapter 9 Differential Equations Q. No.1: Determine order and degree (if defined) of differential equation (y‴)2 + (y″)3 + (y′)4 + y5 = 0 Solution: Given differential equation is (y‴)2 + (y″)3 + (y′)4 + y5 = 0 The highest order derivative present in the differential equation is y‴. Therefore, its order is 3. The given differential equation is a polynomial equation in y‴, y″, and y′. The highest power raised to y‴ is 2. Hence, its degree is 2. Q. No. 2: Verify that the function y = a cos x + b sin x, where, a, b ∈ R is a solution of the differential equation d2y/dx2 + y=0. Solution: The given function is y = a cos x + b sin x ... (1) Differentiating both sides of equation (1) with respect to x, dy/dx = – a sinx + b cos x d2y/dx2 = – a cos x – b sinx LHS = d2y/dx2 + y = – a cos x – b sinx + a cos x + b sin x = 0 = RHS Hence, the given function is a solution to the given differential equation. Q. No. 3: The number of arbitrary constants in the general solution of a differential equation of fourth order is: (A) 0 (B) 2 (C) 3 (D) 4 Solution: We know that the number of constants in the general solution of a differential equation of order n is equal to its order. Therefore, the number of constants in the general equation of the fourth-order differential equation is four. Hence, the correct answer is D. Note: The number of constants in the general solution of a differential equation of order n is equal to zero. Q. No. 4: Form the differential equation representing the family of curves y = a sin (x + b), where a, b are arbitrary constants. Solution: Given, y = a sin (x + b) ... (1) Differentiating both sides of equation (1) with respect to x, dy/dx = a cos (x + b) ... (2) Differentiating again on both sides with respect to x, d2y/dx2 = – a sin (x + b) ... (3) Eliminating a and b from equations (1), (2) and (3), d2y/ dx2 + y = 0 ... (4) The above equation is free from the arbitrary constants a and b. This the required differential equation. Q. No. 5: Find the differential equation of the family of lines through the origin. Solution: Let y = mx be the family of lines through the origin. Therefore, dy/dx = m Eliminating m, (substituting m = y/x) y = (dy/dx) . x or x. dy/dx – y = 0 Q. No. 6: Form the differential equation of the family of circles having a centre on y-axis and radius 3 units. Solution: The general equation of the family of circles having a centre on the y-axis is x2 + (y – b)2 = r2 Given the radius of the circle is 3 units. The differential; equation of the family of circles having a centre on the y-axis and radius 3 units is as below: x2 + (y – b)2 = 32 x2 + (y – b)2 = 9 .....(i) Differentiating (i) with respect to x, 2x + 2(y – b).y′ = 0 ⇒ (y – b). y′ = -x ⇒ (y – b) = -x/y′ .....(ii) Substituting (ii) in (i), x2 + (-x/y′)2 = 9 ⇒ x2[1 + 1/(y′)2] = 9 ⇒ x2 [(y′)2 + 1) = 9 (y′)2 ⇒ (x2 – 9) (y′)2 + x2 = 0 Hence, this is the required differential equation. Q. No. 7: Find the general solution of the differential equation dy/dx =1+y2/1+x2. Solution: Given differential equation is dy/dx =1+y2/1+x2 Since 1 + y2 ≠ 0, therefore by separating the variables, the given differential equation can be written as: dy/1+y2 = dx/1+x2 .....(i) Integrating equation (i) on both sides, tan-1y = tan-1x + C This is the general solution of the given differential equation. Q. No. 8: For each of the given differential equation, find a particular solution satisfying the given condition: dy/dx = y tan x ; y = 1 when x = 0 Solution: dy/dx = y tan x dy/y = tan x dx Integrating on both sides, log y = log (sec x) +C log y = log (C sec x) ⇒ y = C sec x .....(i) Now consider y = 1 when x= 0. 1 = C sec 0 1 = C (1) C = 1 Substituting C = 1 in (i) y = sec x Hence, this is the required particular solution of the given differential equation. Q. No. 9: Find the equation of a curve passing through (1, π/4) if the slope of the tangent to the curve at any point P (x, y) is y/x – cos2(y/x). Solution: According to the given condition, dy/dx = y/x – cos2(y/x) .....(i) This is a homogeneous differential equation. Substituting y = vx in (i), v + (x) dv/dx = v – cos2v ⇒ (x)dv/dx = – cos2v ⇒ sec2v dv = – dx/x By integrating on both the sides, ⇒ ∫sec2v dv = – ∫dx/x ⇒ tan v = – log x + c ⇒ tan (y/x) + log x = c .....(ii) Substituting x = 1 and y = π/4, ⇒ tan (π/4) + log 1 = c ⇒ 1 + 0 = c ⇒ c = 1 Substituting c = 1 in (ii), tan (y/x) + log x = 1 Q. No. 10: Integrating factor of the differential equation (1 – x2)dy/dx – xy = 1 is (A) -x (B) x/ (1 + x2) (C) √(1- x2) (D) ½ log (1 – x2) Solution: Given differential equation is (1 – x2)dy/dx – xy = 1 (1 – x2)dy/dx = 1 + xy dy/dx = (1/1 – x2) + (x/1 – x2)y dy/dx – (x/1 – x2)y = 1/1-x2 This is of the form dy/dx + Py = Q We can get the integrating factor as below: Let 1 – x2 = t Differentiating with respect to x -2x dx = dt -x dx = dt/2 Now, I.F = √t = √(1- x2) Hence, option C is the correct answer. Related Articles Practice Questions For Class 12 Maths Chapter 9 Differential Equations Solve dy/dx + y = 1. Solve (x +1) dy/dx = 2xy In a bank principal increases at the rate of 6% per year. In how many years Rs 2500 double itself. Solve: dy/dx = cos(x+y) Solve the initial value problem: cos(a+b)dy =dx, y(0) = 0. Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is (x2 + y2)/2xy. In culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present? At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, –3). Find the equation of the curve given that it passes through (–2, 1). Solve the differential equation: Given that dy/dx = e-2y and y = 0 when x = 5. Find the value of x when y = 3. Differential equations are mathematical equations that involve derivatives of a function or functions. They describe how a quantity changes over time or space, representing physical, biological, economic, or other systems. Solutions to differential equations provide functions that satisfy the relationships defined by the derivatives.This article explores differential equations, focusing on their types, formulas, solutions, and practice problems. It covers first-order, second-order, homogeneous, and non-homogeneous equations, along with partial differential equations and Practice Questions on Differential Equations both solved and Unsolved.What are Differential Equations?Differential equations are mathematical equations that involve functions and their derivatives. These equations describe the relationship between a function and its rates of change, providing a way to model and solve real-world problems where change is a factor.Important Formulas on Differential EquationsFirst-Order Differential Equation: "dy/dx = f(x, y)"Second-Order Differential Equation: "$d^2y/dx^2 + p(x) * dy/dx + q(x)y = g(x)$"Homogeneous Differential Equation: "dy/dx + P(x)y = 0"Non-Homogeneous Differential Equation: "dy/dx + P(x)y = Q(x)"Partial Differential Equation (PDE): "$du/dt = c^2 * d^2u/dx^2$"Separable Differential Equation: "dy/dx = g(y)h(x)"Practice Questions on Differential Equations - SolvedThese Practice Questions on Differential Equations with solution will help you to understand the concept of Differential Equations and how to solve them. 1: Solve the differential equation: dy/dx = 2x Given the differential equation: dy/dx = 2xTo find y, we integrate both sides with respect to x: Integrating dy/dx with respect to x gives us y: y = ∫2x dxNow, let's integrate 2x with respect to x: ∫2x dx = x2 + CTherefore, the solution to the differential equation dy/dx = 2x is: y = x2 + CHere, C is the constant of integration, which can be determined if initial conditions are provided. 2: Solve the initial value problem: dy/dx = 3x2, y(0) = 2Given the initial value problem: dy/dx = 3x2, y(0) = 2To find y, integrate both sides of the differential equation with respect to x: Integrating dy/dx with respect to x gives us y: y = ∫ 3x^2 dxNow, integrate 3x2 with respect to x: ∫3x2 dx = x3 + CTherefore, the general solution to the differential equation is: y = x3 + CTo find the particular solution that satisfies the initial condition y(0) = 2, substitute x = 0 into the equation: 2 = 03 + C C = 2Thus, the specific solution to the initial value problem dy/dx = 3x2, y(0) = 2 is: y = x3 + 23: Solve the differential equation: dy/dx = exTo find y, integrate both sides with respect to x: Integrating dy/dx with respect to x gives us y: y = ∫ ex dxNow, integrate ex with respect to x: ∫ex dx = ex + CTherefore, the solution to the differential equation dy/dx = ex is: y = ex + CHere, C is the constant of integration, which can be determined if initial conditions are provided. If no initial conditions are given, the solution is typically expressed with C as an arbitrary constant.4: Solve the initial value problem: dy/dx = 1/x, y(1) = 3Given the initial value problem: dy/dx = 1/x, y(1) = 3To find y, integrate both sides of the differential equation with respect to x: Integrating dy/dx with respect to x gives us y: y = ∫1/x dxNow, integrate 1/x with respect to x: ∫1/x dx = ln|x| + CTherefore, the general solution to the differential equation is: y = ln|x| + CTo find the particular solution that satisfies the initial condition y(1) = 3, substitute x = 1 into the equation: 3 = ln|1| + C 3 = 0 + C C = 3Thus, the specific solution to the initial value problem dy/dx = 1/x, y(1) = 3 is: y = ln|x| + 3So, your final answer is: y = ln|x| + 35: Solve the differential equation: dy/dx = (1/2)yThis is a separable differential equation. To solve it, separate the variables: ∫(1/y) dy = ∫(1/2) dxIntegrate both sides: ln|y| = (x/2) + CTo solve for y, exponentiate both sides to eliminate the natural logarithm: |y| = e^((x/2) + C)Simplify the right-hand side: |y| = e(x/2) × eCLet eC be denoted as another constant D: |y| = De(x/2)Since D is a constant, we can drop the absolute value sign: y = De(x/2)Practice Problems on Differential Equations - UnsolvedThese Practice Questions on Differential Equations are to test your understanding of the concept.1. Solve the differential equation: "dy/dx = 3x² - 6".2. Find the general solution of the differential equation: "dy/dx = (2x + 1) / y".3. Solve the initial value problem: "dy/dx = sin(x)", with "y(0) = 1".4. Determine the particular solution of the differential equation: "y' + y = e^x".5. Find the solution to the differential equation: "y' + 2xy = 3x".6. Solve the differential equation: "dy/dx = 2y + x²".7. Determine the general solution to the differential equation: "y' = 4y - 3".8. Solve the initial value problem: "y' = 2/x - y/x", with "y(1) = 1".9. Find the solution to the differential equation: "y' = 1/2 * √(y)".10. Solve the differential equation: "y'' + 4y' + 4y = 0".ConclusionThis article has covered first-order, second-order, homogeneous, and non-homogeneous equations, along with partial differential equations. Each problem is solved in a step-by-step manner, emphasizing integration techniques and initial value problems. The content aims to provide a complete understanding and practical application of differential equations in various contexts. Differential equation questions and answers will help students quickly grasp the fundamentals of the concept. Students can use these questions to quickly summarize the concepts and practice answering them in order to increase their understanding. You can also acquire comprehensive answers to each question to verify your answers. To find out more about differential equations, click here. What are Differential Equations? In calculus, an equation which involves the derivatives of the dependent variable with reference to the independent variable is referred to as a differential equation. The differential equation assists us in presenting a relation between the changing quantity with regard to the change in the other quantity. The derivative represents the rate of change. Be a function with the form: y=f(x), where x is an independent variable, and y is the dependent variable. Also, read: Derivatives. Differential Equation Questions with Solutions First Order Differential Equation: The first-order differential equation includes all linear equations that take the form of derivatives. It only has the first derivative, which is expressed as dy/dx or y′ or f(x, y), where x and y are the two variables. Second-Order Differential Equation: The second-order differential equation is the equation that has second-order derivatives. The second-order differential equation is expressed as d/dx(dy/dx) or f″(x) or d2y/dx2 or y″. Also, read: Order of Differential Equations. 1. What is the order of the differential equation dy/dx + y = 3y2. Solution: Given: dy/dx + y = 3y2 The order of the differential equation is 1st order. In the given differential equation, dy/dx represents the first-order differential equation. Hence, the order of the differential equation dy/dx + y = 3y2 is 1. 2. Find the order of the differential equation y‴ + y″y′ = 3x2. Solution: Given differential equation is y‴ + y″y′ = 3x2. Here, the highest order of the given differential equation is 3. Because, in the given differential equation, y‴ = d3y/dx3, which denotes the third-order differential equation. Therefore, the order of the differential equation y‴ + y″y′ = 3x2 is 3. 3. Find the order and degree of the differential equation y′+5y = 0. Solution: Given: y′ + 5y = 0 The highest order of the derivative is 1. Therefore, order = 1. The degree of the differential equation = power of y′ Hence, degree = 1. Ordinary Differential Equation: In Calculus, the Ordinary Differential Equation (ODE) is defined as an equation which has only one independent variable and it can have one or more derivatives with regard to the variable. An ODE can be either homogeneous or non-homogeneous. Generally, the ODEs can be expressed using the relation which contains one independent variable (let's say x) and the real dependent variable (let's say y), and a few of its derivatives (say, y, y′, y″,...yn) with respect to the independent variable x. Partial Differential Equation: The partial differential equation, also known as PDE, is defined as an equation which involves one or many unknown multi variables with partial derivatives of one or many functions. Generally, in PDEs, the partial derivatives are expressed as ux = ∂u/∂x. 4. Differentiate the function 10x2 with respect to x. Solution: Let the function be "y". Hence, y = 10x2 Now, differentiate the function y = 10x2 with respect to x, and we get dy/dx = 2(10)x [As (d/dx)xn = nxn-1] Hence, dy/dx = y′ = 20x. 5. Find the derivative of sin(3x+5) Solution: Let y = sin (3x+5) As we know, the (d/dx) sin x = cos x. Hence, dy/dx = cos (3x+5). (3) [Since, differentiation of 3x is 3] Thus, dy/dx = 3 cos (3x + 5) Hence, the derivative of sin (3x + 5) is 3 cos (3x+5). 6. Show that x = elog x is true for all the real x? Solution: Given equation: x = elog x. If x = 0, then the equation becomes, 0 = elog 0 But log 0 is not defined, and hence the equation x = elog x is not defined for x = 0. If x < 0, log x is not defined for negative numbers and hence the equation x = elog x is not defined for x < 0. If x > 0, log x is defined for x > 0. Hence, the equation x = e log x is defined for x > 0. This shows that the equation x = elog x is true for all positive values of x. 7. What is the number of arbitrary constants in the general solution of the given differential equation, if its order is 4? Solution: For example, the differential equation of fourth order is y‴″ = 2. Hence, its general solution becomes y = (x4/24) + (Ax3/6) + (Bx2/2) + Cx + D Thus, the number of constants is 4. Therefore, the number of arbitrary constants is 4. In other words, the order of differential equations is equal to the number of arbitrary constants. Since the order of the given differential equation is 4, the number of the arbitrary constants should be 4. 8. Find the number of arbitrary constants in the particular solution of the given differential equation if its order is 3. Solution: As we know, in a particular solution, there should be any arbitrary constants. Therefore, the number of arbitrary constants in the particular solution of the differential equation of third order is 0. 9. Find the integrating factor of the differential equation x(dy/dx) – y = 2x2. Solution: Given the differential equation: x(dy/dx) – y = 2x2 Now, divide both sides of the equation by x, we get (x/x)(dy/dx) – (y/x) = (2x2/x) (dy/dx) – (y/x) = 2x ...(1) Hence, the differential equation is of the form: (dy/dx) + Py = Q ...(2) Comparing the equations (1) and (2), we get P = -1/x, and Q = 2x. We know that the Integrating factor, IF = e∫Pdx Now, substitute the obtained values in the above formula, and we get IF = e∫ (-1/x)dx IF = e– log x As, x log y = log yx, \\begin{array}{l}IF = e^{log x^{-1}}\end{array} \) Hence, IF = x-1 [Since elog x = x] IF = 1/x [As, x-1 = 1/x] Therefore, the integrating factor (IF) of the differential equation x(dy/dx) – y = 2x2 is 1/x. 10. Find the second order derivative of x20. Solution: Assume that y = x20. Now, differentiate the function with respect to x, we get dy/dx = (d/dx)(x20) = 20x19. [Since (d/dx)xn = nxn-1] Again differentiate the above function to find the second derivative. (d2y/dx2) = (d/dx)(20x19) (d2y/dx2) = (20).(19) x18 (d2y/dx2) = 380x18. Hence, the second order derivative of x20 is 380x18. Explore More Articles Practice Questions Solve the following differential equation questions. What is the second order derivative of log x. Find the degree of the differential equation y = x(dy/dx)2 + (dy/dx). What is the general solution of xy(dy/dx) – 1 = 0. Download BYJU'S – The Learning App and stay tuned with us to quickly learn all Maths-related concepts quickly by watching videos.